F-Test

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F- test is used to find out whether the variance of sample A is equal to sample B.

Assumptions:

- The values in both datasets are random and normally distributed
- X1 and X2 are independent

Hypotheses:

H_0: $\sigma_1^2 = \sigma_1^2$ # there is no difference between the variances
H_1: $\sigma_1^2 \neq \sigma_1^2$ # Two Sided Case
H_1: $\sigma_1^2 < \sigma_1^2$ # One Sided Case (Lower Tail)
H_1: $\sigma_1^2 > \sigma_1^2$ # One Sided Case (Upper Tail)

Test Statistics:

```
var.test(X1,X2,alternative="two.sided",conf.level=0.95)
var.test(X1,X2,alternative="less",conf.level=0.95)
var.test(X1,X2,alternative="greater",conf.level=0.95)
```

Decision Rule:

If P-value < α, THEN REJECT H_0, OTHERWISE ACCEPT H_0.

The error rate (α) must be set by the user. (0.05, 0.01 are common values)

Sample of the F-test in R:-

```
# X=read.table("Pizza.txt",header=T)

#  attach(X)

#  X1=Time[Company=="A"]
   X1
[1] 20.4 24.2 15.4 21.4 20.2 18.5 21.5

#  X2=Time[Company=="B"]
  X2
[1] 20.2 16.9 18.5 17.3 20.5
```

Time	*Company*
20.4	A
24.2	A
15.4	A
21.4	A
20.2	A
18.5	A
21.5	A
20.2	B
16.9	B
18.5	B
17.3	B
20.5	B

Test Statistics:

In the following example we can assume that alpha is equal to 0.05

Two Tail Case

```
> var.test(X1,X2,alternative="two.sided",conf.level=0.95)

        F test to compare two variances

data:  X1 and X2
F = 2.8023, num df = 6, denom df = 4, p-value = 0.3378
alternative hypothesis: true ratio of variances is not equal
to 1
95 percent confidence interval:
 0.3046847 17.4502476
sample estimates:
ratio of variances
         2.80228
```

One Sided Case Lower Tail

```
> var.test(X1,X2,alternative="less",conf.level=0.95)

        F test to compare two variances

data:  X1 and X2
F = 2.8023, num df = 6, denom df = 4, p-value = 0.8311
alternative hypothesis: true ratio of variances is less than 1
95 percent confidence interval:
  0.00000 12.70463
sample estimates:
ratio of variances
         2.80228
```

One Sided Case Upper Tail

```
> var.test(X1,X2,alternative="greater",conf.level=0.95)

        F test to compare two variances

data:  X1 and X2
F = 2.8023, num df = 6, denom df = 4, p-value = 0.1689
alternative hypothesis: true ratio of variances is greater than
1
95 percent confidence interval:
 0.4546843       Inf
sample estimates:
ratio of variances
         2.80228
```

As a conclusion of the above results:-

The Two Tail Case
The One Sided Case (Lower Tail)
The One Sided case (Upper Tail)

} The three cases show that P-value is greater than alpha. As a conclusion we accept the Null hypotheses, which states that both samples have equal variances.

The next step is to perform the Two Sample T-test with equal variances to test whether the mean of the two populations varies.