ORIGIN ≡ 0

One Sample t-Test

This test and associated descriptive statistics is designed to test hypotheses about the mean of a population with unknown variance.

Two Tail Case:

ZAR := READPRN("c:/DATA/Biostatistics/ZarEX7.2.txt") **< Zar Example 7.2**

$X := ZAR^{\langle 1 \rangle}$

$n := \text{length}(X)$ $n = 12$

$X_{bar} := \text{mean}(X)$ $X_{bar} = -0.65$

$s := \sqrt{\text{Var}(X)}$ $s = 1.2523$ $s^2 = 1.5682$

ZAR =

	0	1
0	1	1.7
1	2	0.7
2	3	-0.4
3	4	-1.8
4	5	0.2
5	6	0.9
6	7	-1.2
7	8	-0.9
8	9	-1.8
9	10	-1.4
10	11	-1.8
11	12	-2

Assumptions:

- **Observed values $X_1, X_2, X_3, \ldots X_n$ are a random sample from $\sim N(\mu,\sigma^2)$.**
- **Variance σ^2 of the population is *unknown*.**

^ Note: this test is reasonably robust for deviations from $\sim N(\mu,\sigma^2)$.

Hypotheses:

$\mu_0 := 0$ **< Let $\mu_0 = 0$**

H_0: $\mu = \mu_0$ **< μ_0 is a specified value for μ**

H_1: $\mu \neq \mu_0$ **< Two sided test**

Test Statistic:

$$t := \frac{X_{bar} - \mu_0}{\frac{s}{\sqrt{n}}} \qquad t = -1.7981$$

< t is the normalized distance between means X_{bar} and $\mu_0 = 0$

Sampling Distribution:

If Assumptions hold and H_0 is true, then $t \sim t_{(n-1)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C_1 := qt\left(\frac{\alpha}{2}, n-1\right)$ $C_1 = -2.201$

$C_2 := qt\left(1 - \frac{\alpha}{2}, n-1\right)$ $C_2 = 2.201$

< Critical values found by the inverse cumulative t function. Note: two critical values are found one for each tail of the distribution. Because the t distribution is symmetrical, the Critical values are of equal absolute magnitude.

Decision Rule:

IF $t < C_1$ or IF $t > C_2$, THEN REJECT H_0, OTHERWISE ACCEPT H_0

IF $|t| > |C|$, THEN REJECT H_0, OTHERWISE ACCEPT H_0

$t = -1.7981$ $C_1 = -2.201$ $C_2 = 2.201$ **< Therefore DO NOT Reject H_0**

Probability Value:

IF $t \le 0$ then: $P := 2 \cdot pt(t, n-1)$ $P = 0.099637$ **< this value** **< Rosner 2006 Eq. 7.11 p. 241**

IF $t > 0$ then: $P := 2 \cdot (1 - pt(t, n-1))$ $P = 1.9004$

Confidence Interval for the Mean:

$C := |C_1|$ $C = 2.201$

$$CI := \left(X_{bar} - C \cdot \frac{s}{\sqrt{n}} \quad X_{bar} + C \cdot \frac{s}{\sqrt{n}}\right) \qquad CI = (-1.4457 \quad 0.1457) \qquad X_{bar} = -0.65$$

Two Tail Case:

ZAR := READPRN("c:/DATA/Biostatistics/ZarEX7.1.txt") **< Zar Example 7.1**

$X := ZAR^{\langle 1 \rangle}$

$n := \text{length}(X)$ $n = 25$

$X_{bar} := \text{mean}(X)$ $X_{bar} = 25.028$

$s := \sqrt{\text{Var}(X)}$ $s = 1.3418$ $s^2 = 1.8004$

ZAR =

	0	1
0	1	25.8
1	2	24.6
2	3	26.1
3	4	22.9
4	5	25.1
5	6	27.3
6	7	24
7	8	24.5
8	9	23.9
9	10	26.2
10	11	24.3
11	12	24.6
12	13	23.3
13	14	25.5

Assumptions:

- Observed values $X_1, X_2, X_3, \ldots X_n$ are a random sample from $\sim N(\mu,\sigma^2)$.

- Variance σ^2 of the population is *unknown.*

Hypotheses:

$\mu_0 := 24.3$ **< Let $\mu_0 = 24.3$**

H_0: $\mu = \mu_0$ **< μ_0 is a specified value for μ**

H_1: $\mu \neq \mu_0$ **< Two sided test**

Test Statistic:

$$t := \frac{X_{bar} - \mu_0}{\frac{s}{\sqrt{n}}}$$ $t = 2.7128$ **< t is the normalized distance between means Xbar and μ_0**

Sampling Distribution:

If Assumptions hold and H_0 is true, then $t \sim t_{(n-1)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C_1 := qt\left(\frac{\alpha}{2}, n-1\right)$ $C_1 = -2.0639$

$C_2 := qt\left(1 - \frac{\alpha}{2}, n-1\right)$ $C_2 = 2.0639$

< Critical values found by the inverse cumulative t function. Note: two critical values are found one for each tail of the distribution. Because the t distribution is symmetrical, the Critical values are of equal absolute magnitude.

Decision Rule:

IF $t < C_1$ or IF $t > C_2$, THEN REJECT H_0, OTHERWISE ACCEPT H_0

IF $|t| > |C|$, THEN REJECT H_0, OTHERWISE ACCEPT H_0

$t = 2.7128$ $C_1 = -2.0639$ $C_2 = 2.0639$ **< Therefore REJECT H_0**

Probability Value:

IF $t \le 0$ **then:** $P := 2 \cdot pt(t, n-1)$ $P = 1.9879$

IF $t > 0$ then: $P := 2 \cdot (1 - pt(t, n-1))$ $P = 0.012146$ **< this value**

< Rosner 2006 Eq. 7.11 p. 241

Confidence Interval for the Mean:

$C := |C_1|$ $C = 2.0639$

$$CI := \left(X_{bar} - C \cdot \frac{s}{\sqrt{n}} \quad X_{bar} + C \cdot \frac{s}{\sqrt{n}}\right)$$ $CI = (24.4741 \quad 25.5819)$ $X_{bar} = 25.028$

One Tail Case (Lower Tail):

ZAR := READPRN("c:/DATA/Biostatistics/ZarEX7.3.txt") **< Zar Example 7.3**

$X := ZAR^{\langle 1 \rangle}$

$n := \text{length}(X)$ $n = 12$

$X_{bar} := \text{mean}(X)$ $X_{bar} = -0.6083$

$s := \sqrt{\text{Var}(X)}$ $s = 0.6331$ $s^2 = 0.4008$

ZAR =

	0	1
0	1	0.2
1	2	-0.5
2	3	-1.3
3	4	-1.6
4	5	-0.7
5	6	0.4
6	7	-0.1
7	8	0
8	9	-0.6
9	10	-1.1
10	11	-1.2
11	12	-0.8

Assumptions:

- **Observed values X_1, X_2, X_3, ... X_n are a random sample from $\sim N(\mu,\sigma^2)$.**
- **Variance σ^2 of the population is *unknown.***

Hypotheses:

$\mu_0 := 0$ **< Let $\mu_0 = 0$**

H_0: $\mu \geq \mu_0$ **< μ_0 is a specified value for μ**

H_1: $\mu < \mu_0$ **< One sided test**

Test Statistic:

$$t := \frac{X_{bar} - \mu_0}{\frac{s}{\sqrt{n}}}$$ $t = -3.3285$ **< t is the normalized distance between means Xbar and μ_0**

Sampling Distribution:

If Assumptions hold and H_0 is true, then $t \sim t_{(n-1)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C := qt(\alpha, n-1)$ $C = -1.7959$ **< Critical value is found by the inverse cumulative t function. Only one Critical value is found.**

Decision Rule:

IF t < C, THEN REJECT H_0, OTHERWISE ACCEPT H_0

$t = -3.3285$ $C = -1.7959$ **< Therefore REJECT H_0**

Probability Value:

$P := pt(t, n-1)$ $P = 0.003364$ **< Rosner 2006 p. 237**

Confidence Interval for the Mean:

$$CI := X_{bar} + |C| \cdot \frac{s}{\sqrt{n}}$$ **minus infinity to** $CI = -0.2801$ $X_{bar} = -0.6083$

^ upper limit

Other One Tail Case (Upper Tail):

ZAR := READPRN("c:/DATA/Biostatistics/ZarEX7.4.txt") **< Zar Example 7.4**

ZAR =

	0	1
0	1	42.7
1	2	43.4
2	3	44.6
3	4	45.1
4	5	45.6
5	6	45.9
6	7	46.8
7	8	47.6

$X := ZAR^{\langle 1 \rangle}$

$n := \text{length}(X)$ $n = 8$

$X_{bar} := \text{mean}(X)$ $X_{bar} = 45.2125$

$s := \sqrt{\text{Var}(X)}$ $s = 1.6401$ $s^2 = 2.6898$

Assumptions:

- Observed values X_1, X_2, X_3, ... X_n are a random sample from ~N(μ,σ^2).

- Variance σ^2 of the population is *unknown*.

Hypotheses:

$\mu_0 := 45$ **< Let μ_0= 45**

H_0: $\mu \le \mu_0$ **< μ_0 is a specified value for μ**

H_1: $\mu > \mu_0$ **< One sided test**

Test Statistic:

$$t := \frac{X_{bar} - \mu_0}{\frac{s}{\sqrt{n}}}$$ $t = 0.3665$ **< t is the normalized distance between means Xbar and μ_0**

Sampling Distribution:

If Assumptions hold and H_0 is true, then t ~$t_{(n-1)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C := qt(1 - \alpha, n - 1)$ $C = 1.8946$ **< Critical value is found by the inverse cumulative t function. Only one Critical value is found.**

Decision Rule:

IF t > C, THEN REJECT H_0, OTHERWISE ACCEPT H_0

$t = 0.3665$ $C = 1.8946$ **< Therefore DO NOT reject H_0**

Probability Value:

$P := 1 - pt(t, n - 1)$ $P = 0.3624$ **< Rosner 2006 p. 237**

Confidence Interval for the Mean:

$$CI := X_{bar} - |C| \cdot \frac{s}{\sqrt{n}}$$ $CI = 44.1139$ **to infinity** $X_{bar} = 45.2125$

^ lower limit

Prototype in R:

```
#t.test IN R
#TO FIND INFORMATION ON t.test()
?t.test

#ZAR EXAMPLE 7.2
ZAR=read.table("c:/DATA/Biostatistics/ZarEX7.2.txt")
ZAR
attach(ZAR)
X=RATwtchg
t.test(X,alternative="two.sided", mu=0,conf.level=0.95)
```

```
 One Sample t-test

data:  X
t = -1.7981, df = 11, p-value = 0.09964
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -1.4456548  0.1456548
sample estimates:
mean of x
   -0.65
```

```
#ZAR EXAMPLE 7.1
ZAR=read.table("c:/DATA/Biostatistics/ZarEX7.1.txt")
ZAR
attach(ZAR)
X=crabtemp
t.test(X,alternative="two.sided", mu=24.3,conf.level=0.95)
```

```
 One Sample t-test

data:  X
t = 2.7128, df = 24, p-value = 0.01215
alternative hypothesis: true mean is not equal to 24.3
95 percent confidence interval:
 24.47413 25.58187
sample estimates:
mean of x
  25.028
```

```
#ZAR EXAMPLE 7.3
ZAR=read.table("c:/DATA/Biostatistics/ZarEX7.3.txt")
ZAR
attach(ZAR)
X=wtchg
t.test(X,alternative="less", mu=0,conf.level=0.95)
#NOTE: R REPORTS A ONE-WAY CI
```

```
 One Sample t-test

data:  X
t = -3.3285, df = 11, p-value = 0.003364
alternative hypothesis: true mean is less than 0
95 percent confidence interval:
      -Inf -0.2801098
sample estimates:
 mean of x
-0.6083333
```

```
#ZAR EXAMPLE 7.4
ZAR=read.table("c:/DATA/Biostatistics/ZarEX7.4.txt")
ZAR
attach(ZAR)
X=distime
t.test(X,alternative="greater", mu=45,conf.level=0.95)
#NOTE: R REPORTS A ONE-WAY CI
```

```
 One Sample t-test

data:  X
t = 0.3665, df = 7, p-value = 0.3624
alternative hypothesis: true mean is greater than 45
95 percent confidence interval:
 44.11393      Inf
sample estimates:
mean of x
  45.2125
```