ORIGIN ≡ 0

Estimating Power and Sample Size for a One Sample t-Test

Pilot studies are often run in advance of collecting data for major statistical analyses. These studies are used to determine the POWER of an analysis - i.e., the ability of the analysis to satisfactorily lead to rejection of the Null Hypothesis and determining sufficient Sample Size to support sufficient power.

Assumptions:

- **Observed values $X_1, X_2, X_3, \ldots X_n$ are a random sample from $\sim N(\mu,\sigma^2)$.**
- **Variance σ^2 of the population is *unknown.***

Estimating Sample Size for Desired Confidence Interval Width:

ZAR := READPRN("c:/DATA/Biostatistics/ZarEX7.3.txt") **< Zar Example 7.3**

$X := ZAR^{\langle 1 \rangle}$

$n := \text{length}(X)$ $n = 12$

$X_{bar} := \text{mean}(X)$ $X_{bar} = -0.6083$

$s := \sqrt{\text{Var}(X)}$ $s = 0.6331$ $s^2 = 0.400833$

ZAR =

	0	1
0	1	0.2
1	2	-0.5
2	3	-1.3
3	4	-1.6
4	5	-0.7
5	6	0.4
6	7	-0.1
7	8	0
8	9	-0.6
9	10	-1.1
10	11	-1.2
11	12	-0.8

Confidence Intervals (CI) are the primary means to estimate a population mean μ from sample data. The width of a CI can be controlled by sample size N for a given population variance σ^2. Zar 2010 offers an interative procedure for estimating a minimum suitable sample size N. Note that N is the sample size we wish to estimate, not the sample size n of our pilot sample.

Desired CI half width:

$d := 0.25$ **< This is set as desired. Note that 2d is the CI width.**

Desired Type 1 Error level:

$\alpha := 0.05$ **< Type 1 error α**

Initial Guess of Sample Size:

$N_0 := 40$ **< Initial guess of sample sample size N needed.**

Iterative Calculation:

$$N_1 := \frac{s^2 \cdot qt\left(\frac{\alpha}{2}, N_0 - 1\right)^2}{d^2} \qquad N_1 = 26.23873$$

< This process is continued until the value of N_i stabilizes. Then use the next largest interger value as an estimate of minimum sample size needed.

$$N_2 := \frac{s^2 \cdot qt\left(\frac{\alpha}{2}, N_1 - 1\right)^2}{d^2} \qquad N_2 = 27.17735$$

^ Note: input into the qt() function depends on whether the alternative hypotheses involve a one-sided or two-sided test. This example is a two-sided test. For a one-sided test, use α instead of $\alpha/2$.

Prototype in R:

```
#POWER & SAMPLE SIZE CALCULATIONS

#ESTIMATING SAMPLE SIZE FOR CI
#ZAR EXAMPLE 7.3
ZAR=read.table("c:/DATA/Biostatistics/ZarEX7.3.txt")
ZAR
attach(ZAR)
X=wtchg
s=sqrt(var(X))  #sample variance
#SET THE FOLLOWING VALUES AS DESIRED:
d=0.25
N0=40
alpha=0.05
#ITERATE THE FOLLOWING UNTIL N IS STABILIZED:
N1=(s^2*(qt(alpha/2,N0-1))^2)/d^2
N1
N2=(s^2*(qt(alpha/2,N1-1))^2)/d^2
N2
N3=(s^2*(qt(alpha/2,N2-1))^2)/d^2
N3
N4=(s^2*(qt(alpha/2,N3-1))^2)/d^2
N4
N5=(s^2*(qt(alpha/2,N4-1))^2)/d^2
N5
```

Estimating Sample Size for a One Sample t-Test:

This estimation differs from the one above in being specifically tied to a distance δ defined by the alternative H_0 and H_1 hypotheses of a one-sample t-test.

ZAR := READPRN("c:/DATA/Biostatistics/ZarEX7.2.txt") **< Zar Example 7.2**

$X := ZAR^{\langle 1 \rangle}$

$n := \text{length}(X)$ $n = 12$

$X_{bar} := \text{mean}(X)$ $X_{bar} = -0.65$

$s := \sqrt{\text{Var}(X)}$ $s = 1.2523$ $s^2 = 1.5682$

ZAR =

	0	1
0	1	1.7
1	2	0.7
2	3	-0.4
3	4	-1.8
4	5	0.2
5	6	0.9
6	7	-1.2
7	8	-0.9
8	9	-1.8
9	10	-1.4
10	11	-1.8
11	12	-2

Hypotheses:

$\mu_0 := 0$ **< Let $\mu_0 = 0$**

H_0: $\mu = \mu_0$ **< μ_0 is a specified value for μ**

H_1: $\mu \neq \mu_0$ **< Two sided test**

Desired Precision δ:

$\delta := 1.0$ **< Set as desired for precision in estimating μ-μ_0. We want to reject H_0 if $|\mu-\mu_0| > \delta$**

Desired Type 1 & 2 Error levels:

$\alpha := 0.05$ **< Type 1 error α**

$\beta := 0.10$ **< Type 2 error β**

Initial Guess of Sample Size:

$N_0 := 20$ **< Initial guess of sample sample size N needed.**

Iterative Calculation:

$$N_1 := \frac{s^2}{\delta^2}\cdot\left(qt\left(\frac{\alpha}{2}, N_0 - 1\right) + qt\left(\beta, N_0 - 1\right)\right)^2 \qquad N_1 = 18.35015$$

$$N_2 := \frac{s^2}{\delta^2}\cdot\left(qt\left(\frac{\alpha}{2}, N_1 - 1\right) + qt\left(\beta, N_1 - 1\right)\right)^2 \qquad N_2 = 18.54504$$

< This process is continued until the value of N_i stabilizes. Then use the next largest interger value as an estimate of minimum sample size needed.

^ Note: input into the qt() function depends on whether the alternative hypotheses involve a one-sided or two-sided test. This example is a two-sided test. For a one-sided test, use α instead of $\alpha/2$.

Prototype in R:

```
#ESTIMATING SAMPLE SIZE FOR ONE SAMPLE T-TEST
#ZAR EXAMPLE 7.2
ZAR=read.table("c:/DATA/Biostatistics/ZarEX7.2.txt")
ZAR
attach(ZAR)
X=RATwtchg
s=sqrt(var(X))
#SET THE FOLLOWING VALUES AS DESIRED:
delta=1.0
alpha=0.05
beta=0.10
N0=20
```

```
#ITERATE THE FOLLOWING UNTIL N IS STABILIZED:
N1=(s^2/delta^2)*(qt(alpha/2,N0-1)+qt(beta,N0-1))^2
N1
N2=(s^2/delta^2)*(qt(alpha/2,N1-1)+qt(beta,N1-1))^2
N2
N3=(s^2/delta^2)*(qt(alpha/2,N2-1)+qt(beta,N2-1))^2
N3
N4=(s^2/delta^2)*(qt(alpha/2,N3-1)+qt(beta,N3-1))^2
N4
N5=(s^2/delta^2)*(qt(alpha/2,N4-1)+qt(beta,N4-1))^2
N5
```

Estimating Detectable Difference of a given Sample Size for a One Sample t-Test:

For a given sample with size n, one can estimate $\delta = \mu-\mu_0$ directly.

Desired Sample Size:

$N := 25$ **< set for desired sample size N**

Desired Type 1 & 2 Error levels:

$\alpha := 0.05$ **< Type 1 error α**

$\beta := 0.10$ **< Type 2 error β**

Calculation:

$$\delta := \sqrt{\frac{s^2}{N}}\cdot\left(qt\left(\frac{\alpha}{2}, N - 1\right) + qt\left(\beta, N - 1\right)\right) \qquad \delta = -0.8469694$$

< Note: sign of δ doesn't matter

^ Note: input into the qt() function depends on whether the alternative hypotheses involve a one-sided or two-sided test. This example is a two-sided test. For a one-sided test, use α instead of $\alpha/2$.

Prototype in R:

```
#ESTIMATING DETECTABLE DIFFERENCE GIVEN N
#FOR ONE SAMPLE t-TEST
#SET THE FOLLOWING VALUES AS DESIRED:
N=25
alpha=0.05
beta=0.10
delta=sqrt(s^2/N)*(qt(alpha/2,N-1)+qt(beta,N-1))
delta
```

Estimating POWER of a One Sample t-test:

POWER (1-β) of a test is the probability of properly rejecting H_0 when it is false. It is the converse of Type 2 error β. We would like POWER to be as high as possible.

Desired Sample Size:

$N := 12$ **< set for desired sample size N**

Desired Precision δ:

$\delta := 1.0$ **< Set as desired for precision in estimating $\mu-\mu_0$. We want to reject H_0 if $|\mu-\mu_0| > \delta$**

Desired Type 1 Error level:

$\alpha := 0.05$ **< Set Type 1 error α**

Calculation:

$$B := \frac{\delta}{\sqrt{\frac{s^2}{N}}} - \left| qt\left(\frac{\alpha}{2}, N-1\right)\right| \qquad B = 0.5652711$$

^ Note: absolute value used here to allow use of standard qt() function whereas Zar uses a *partial* table that only has positive values.

$POWER := pt(B, N-1)$ $POWER = 0.7083827$ **< Exact calculation using pt() function**

$POWER_N := pnorm(B, 0, 1)$ $POWER_N = 0.7140553$ **< Approximate calculation using pnorm() function assuming s=σ.**

^ POWER = (1-β)
Note use of the probability functions.

Prototype in R:

```
#ESTIMATING POWER OF ONE SAMPLE t-TEST
#SET THE FOLLOWING VALUES AS DESIRED:
N=12
delta=1.0
alpha=0.05

B=(delta/sqrt(s^2/N))-abs(qt(alpha/2,N-1))
B
POWER=pt(B,N-1)
POWER
POWERN=pnorm(B,0,1)
POWERN
```