ORIGIN ≡ 0

One Sample χ^2 Test of Variance for a Normal Distribution

This test is designed to test hypotheses about whether the variance of a population σ^2 is statistically equivalent to a specified value σ_0^2.

ZAR := READPRN("c:/DATA/Biostatistics/ZarEX7.4.txt") **< Zar Example 7.4**

$X := ZAR^{\langle 1 \rangle}$

$n := \text{length}(X)$ $n = 8$

$X_{bar} := \text{mean}(X)$ $X_{bar} = 45.2125$

$s := \sqrt{\text{Var}(X)}$ $s = 1.6401$ $s^2 = 2.68982143$

ZAR =

	0	1
0	1	42.7
1	2	43.4
2	3	44.6
3	4	45.1
4	5	45.6
5	6	45.9
6	7	46.8
7	8	47.6

Assumptions:

- **Observed values X_1, X_2, X_3, ... X_n are a random sample from ~N(μ,σ^2).**
  Note that this requirement is critical and not robust, thus limiting this test's usefulness.
- **Variance σ^2 of the population is *unknown.***

Hypotheses:

$\sigma_0 := \sqrt{1.5}$ **< Set value of σ_0 as desired**

H_0: $\sigma^2 = \sigma_0^2$ **< σ_0^2 is a specified value for σ^2**

H_1: $\sigma^2 \neq \sigma_0^2$ < **Two Sided Case**

H_1: $\sigma^2 < \sigma_0^2$ < **One Sided Case Lower Tail)**

H_1: $\sigma^2 > \sigma_0^2$ < **One Sided Case (Upper Tail)**

Test Statistic:

$Xsq := \frac{(n-1) \cdot s^2}{\sigma_0^2}$ $Xsq = 12.5525$ **< population corrected ratio of observed sample variance and hypothesized variance**

Sampling Distribution:

If Assumptions hold and H_0 is true, then Xsq ~ $\chi^2_{(n-1)}$

Critical Values of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C_1 := \text{qchisq}\left(\frac{\alpha}{2}, n-1\right)$ $C_1 = 1.689869$ **< Two sided lower Critical Value**

$C_2 := \text{qchisq}\left(1 - \frac{\alpha}{2}, n-1\right)$ $C_2 = 16.01276$ **< Two sided upper Critical Value**

$C_3 := \text{qchisq}(\alpha, n-1)$ $C_3 = 2.16735$ **< One sided lower Critical Value**

$C_4 := \text{qchisq}(1-\alpha, n-1)$ $C_4 = 14.06714$ **< One sided upper Critical Value**

Decision Rules:

IF Xsq < C_1 or Xsq > C_2 , THEN REJECT H_0, OTHERWISE ACCEPT H_0 **< Two sided case**
IF Xsq < C_3, THEN REJECT H_0, OTHERWISE ACCEPT H_0 **< One sided case lower tail**
IF Xsq > C_4, THEN REJECT H_0, OTHERWISE ACCEPT H_0 **< One sided case upper tail**

Xsq = 12.5525

Probability Values:

$P := 2 \cdot (1 - \text{pchisq}(Xsq, n-1))$ P = 0.1676 **< Two sided case**

$P := \text{pchisq}(Xsq, n-1)$ P = 0.9162 **< One sided case lower tail**

$P := 1 - \text{pchisq}(Xsq, n-1)$ P = 0.0838 **< One sided case upper tail**

Confidence Intervals for σ^2:

$s^2 = 2.6898$

$CI := \left[\frac{(n-1) \cdot s^2}{C_2} \quad \frac{(n-1) \cdot s^2}{C_1} \right]$ CI = (1.17586 11.14213) **< Two sided case**

$CI_L := \frac{(n-1) \cdot s^2}{C_3}$ **minus infinity to** $CI_L = 8.687453$ **< One sided case lower tail**

$CI_U := \frac{(n-1) \cdot s^2}{C_4}$ $CI_U = 1.338492$ **to infinity** **< One sided case upper tail**

^ Note: I generally find two-side confidence intervals more meaningful.

Prototype in R:

```
#TEST OF VARIANCE FOR A NORMAL DISTRIBUTION
#ZAR EXAMPLE 7.4
ZAR=read.table("ZarEX7.4.txt")
ZAR
attach(ZAR)
X=distime
n=length(X)
s=sqrt(var(X))
#USING FUNCTION varTest() in {EnvStats}:
library(EnvStats)
varTest(X,sigma.squared = 1.5,alternative="two.sided",conf.level=0.95)
```

The R script associated with this test includes direct calculation "by hand" because I couldn't find a function in R that would do this. However, Mian Li has come to our rescue! Listed below is the function varTest() found in package {EnvStats}.

```
>varTest(X,sigma.squared = 1.5,alternative="two.sided",conf.level=0.95)
Results of Hypothesis Test
--------------------------
Null Hypothesis:                 variance = 1.5
Alternative Hypothesis:          True variance is not equal to 1.5
Test Name:                       Chi-Squared Test on Variance
Estimated Parameter(s):          variance = 2.689821
Data:                            X
Test Statistic:                  Chi-Squared = 12.5525
Test Statistic Parameter:        df = 7
P-value:                         0.1675838
95% Confidence Interval:         LCL =  1.175859
                                 UCL = 11.142135
```