ORIGIN ≡ 0

Wilcoxon Rank-Sum Test
Mann-Whitney Test

These fully equivalent procedures are the nonparametric analog to the two-sample t-test. They are applied when analyzing independent samples from two populations without assuming an underlying Normal distribution for each. Thus they may be applied to most/all situations one might normally apply to a parametric solution, but with fewer assumptions and less power.

Assumptions:

- **Observed values $X_{1,1}$, $X_{1,2}$, $X_{1,3}$, ... $X_{1,n1}$ are a random sample**
  Observed values $X_{2,1}$, $X_{2,2}$, $X_{2,3}$, ... $X_{2,n2}$ are a random sample.
- **Variables X_1's and X_2 are independent.**
- **Underlying distributions are continuous.**
- **Measurement scale is at least ordinal - i.e, data can be ranked.**

Zar Example 8.11:

0	height	sex	rank
1	193	male	1
2	188	male	2
3	185	male	3
4	183	male	4
5	180	male	5
6	175	male	7
7	170	male	9
8	178	female	6
9	173	female	8
10	168	female	10
11	165	female	11
12	163	female	12

Hypotheses:

H_0: $\Delta = 0$ < No population ordinal difference in median

H_1: $\Delta \neq 0$ < **Two sided test**

Criterion for Normal Approximation:

- **IF** $(n_1 \geq 10) \wedge (n_2 \geq 10)$ **THEN Normal Approximation may be used**
  OTHERWISE use Special Tables e.g., Zar 2010 Appendix B-11 or Rosner 2006 Table 11 in Appendix

$n := 12$

$n_1 := 7$

$n_2 := 5$

Normal Approximation:

Rank Data and Sum:

- **Pool Data and Rank observations.**
- **Compute Rank Sum (RS_1 or RS_2) of one population (doesn't matter which).**

$RS_1 := 1 + 2 + 3 + 4 + 5 + 7 + 9$ $RS_1 = 31$ **< rank sum for males**

$RS_2 := 6 + 8 + 10 + 11 + 12$ $RS_2 = 47$ **< rank sum for females**

Wilcoxon Test Statistic T:

IF RS_1 <> $n_1(n_1+n_2+1)/2$ AND there are NO ties THEN:

$$n_1 \cdot \frac{(n_1 + n_2 + 1)}{2} = 45.5$$

$$T_1 := \frac{\left[\left|RS_1 - \frac{n_1 \cdot (n_1 + n_2 + 1)}{2}\right| - \frac{1}{2}\right]}{\sqrt{\left(\frac{n_1 \cdot n_2}{12}\right) \cdot (n_1 + n_2 + 1)}}$$

$T_1 = 2.2736$ **< applies**

IF RS_1 <> $n_1(n_1+n_2+1)/2$ AND there ARE ties THEN:

$$T_2 := \frac{\left[\left|RS_1 - \frac{n_1 \cdot (n_1 + n_2 + 1)}{2}\right| - \frac{1}{2}\right]}{\sqrt{\left(\frac{n_1 \cdot n_2}{12}\right) \cdot \left[n_1 + n_2 + 1 - \sum_i \frac{t_i \cdot \left[(t_i)^2 - 1\right]}{(n_1 + n_2) \cdot (n_1 + n_2 - 1)}\right]}}$$

$T_2 = ▮$

where:

t = number of tied individuals in each class or group.

i = is used to sum across all classes or groups.

IF RS_1 = $n_1(n_1+n_2+1)/2$ THEN:

$T_3 := 0$ $T_3 = 0$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C := \text{qnorm}\left(1 - \frac{\alpha}{2}, 0, 1\right)$ $C = 1.96$

Decision Rule:

IF T > C THEN REJECT H_0, OTHERWISE ACCEPT H_0

$T_1 = 2.2736$

Probability Value:

$P := 2 \cdot \left(1 - \text{pnorm}(T_1, 0, 1)\right)$ $P = 0.023$

Mann-Whitney Test Statistic U:

$U := n_1 \cdot n_2 + \frac{n_1 \cdot (n_1 + 1)}{2} - RS_1$ $U = 32$

^ This statistic may be compared with Zar 2010 Appendix B-11.

Prototype in R:

```
#WILCOXON RANK-SUM TEST
#MANN-WHITNEY TEST

ZAR=read.table("c:/DATA/Biostatistics/ZarEX8.11.txt")
ZAR
attach(ZAR)

wilcox.test(height~sex,paired=FALSE,exact=T,alternative="two.sided")
```

^ according to the documentation for wilcox.test() explicit calculation of the test statistic W is made if the samples contain less than 50 values and there are no ties. Otherwise a Normal Approximation is used.

```
        Wilcoxon rank sum test

data:  height by sex
W = 3, p-value = 0.01768
alternative hypothesis: true location shift is not equal to 0
```

^Results show a small (but not the same) P value as expected since this dataset didn't qualify for the Normal approximation. Note also that R's statistic W doesn't match! See R's documentation about this... and below...

$W := 3$ **< Reported statistic W from R above.**

$cf := \frac{n_1 \cdot (n_1 + 1)}{2}$ $cf = 28$ **< correction factor indicated in documentation**

$W + cf = 31$ $RS_1 = 31$ **< W + cf is the same as our RS_1**