ORIGIN ≡ 1

Linear Contrasts in One-Way ANOVA with Fixed Effects Model

When the omnibus F-Test for ANOVA rejects the null hypothesis that all α_i's = 0 (or all μ_i's are equal), then one usually wants to determine *specifically which* of the α_i's is not equal to 0 (or which mean μ_i is different from the others). This test approach is a generalization of the t-test comparing pairs of means. Here any *linear combination*:

$$L = c_1 Xbar_1 + c_2 Xbar_2 + c_3 Xbar_3 + \ldots + c_k Xbar_k$$

can be tested where X_1, X_2, X_3 .. X_k represent samples derived from different populations 1,2, 3 ...k, and the c_i's are *coefficients* of the linear combination that must add to zero. As can be readily imagined, multiple linear contrasts are possible. In fact, pairwise comparisons of means are a special case of linear constrasts in which two coefficients have equal value but different sign (e.g., c_1= -1 & c_2= 1) and all other c_i's are zero. Linear constrasts may therefore be tested either singly using t-tests, or with multiple comparison procedures (MCP).

Data Structure:

k groups with not necessarily the same numbers of observations and different means.

Let index i,j indicate the ith column (treatment class) and jth row (object).

One-Way ANOVA					
	Treatment Classes:				
Objects (Replicates)	#1	#2	#3	...	#k
1					
2					
3					
...					
n	n1	n2	n3		nk
means:	X1bar	X2bar	X3bar	...	Xkbar

```
ZAR=read.table("c:/DATA/Biostatistics/ZarEX11.1R.txt")
ZAR
attach(ZAR)
Y=conc
X=factor(code)
options(digits=12)

#DESCRIPTIVE STATISTICS:
N=length(Y)
N
k=5
#MEANS:
tapply(Y,ZAR[,-1],mean)
#NUMBERS:
tapply(Y,ZAR[-1],length)
#STANDARD DEVIATIONS:
tapply(Y,ZAR[-1],sd)

#GENERATING ANOVA TABLE:
anova(lm(Y~X))
```

Zar Example 11.1

$k := 5$ **< number of classes**

$N := 30$ **< total number of cases**

$$X_{bar} := \begin{pmatrix} 32.0833333333 \\ 40.2333333333 \\ 43.9166666667 \\ 41.1000000000 \\ 58.3000000000 \end{pmatrix} \quad n := \begin{pmatrix} 6 \\ 6 \\ 6 \\ 6 \\ 6 \end{pmatrix} \quad s := \begin{pmatrix} 3.20525610001 \\ 2.53034911952 \\ 3.14478404134 \\ 3.66606055596 \\ 3.03644529014 \end{pmatrix}$$

^ descriptive statistics derived from R

```
of Variance Table

Response: Y
          Df       Sum Sq      Mean Sq  F value       Pr(>F)
X          4 2191.7286667  547.9321667 55.65475 4.3617e-12 ***
Residuals 25  246.1300000    9.8452000
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

One-Way ANOVA Table:

Values taken from R's ANOVA table:

Source:	**SS**	**df**		**MS**	
Between	$SS_B := 2191.7286667$	$df_B := k - 1$	$df_B = 4$	$MS_B := \frac{SS_B}{df_B}$	$MS_B = 547.9322$
Within	$SS_W := 246.1300000$	$df_W := N - k$	$df_W = 25$	$MS_W := \frac{SS_W}{df_W}$	$MS_W = 9.8452$
TOTAL	$SS_T := SS_B + SS_W$ $SS_T = 2437.8587$				

Linear Combinations:

Combinations from Zar 2010 Example 11.5.

combination one:

$$X_{bar} = \begin{pmatrix} 32.0833 \\ 40.2333 \\ 43.9167 \\ 41.1 \\ 58.3 \end{pmatrix}$$

$$c1 := \begin{pmatrix} 0 \\ \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \\ -1 \end{pmatrix} \qquad L_1 := \sum_{i=1}^{k} c1_i \cdot X_{bar_i} \qquad L_1 = -16.55$$

$$0 \cdot 32.0833333 + \frac{1}{3} \cdot 40.2333333 + \frac{1}{3} \cdot 43.9166667 + \frac{1}{3} \cdot 41.1 - 1 \cdot 58.3 = -16.55$$

combination two:

$$c2 := \begin{pmatrix} 1 \\ -\frac{1}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \\ 0 \end{pmatrix} \qquad L_2 := \sum_{i=1}^{k} c2_i \cdot X_{bar_i} \qquad L_2 = -9.6667$$

$$1 \cdot 32.0833333 - \frac{1}{3} \cdot 40.2333333 - \frac{1}{3} \cdot 43.9166667 - \frac{1}{3} \cdot 41.1 + 0 \cdot 58.3 = -9.6667$$

combination three:

$$c3 := \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{3} \\ -\frac{1}{3} \\ -\frac{1}{3} \\ \frac{1}{2} \end{pmatrix} \qquad L_3 := \sum_{i=1}^{k} c3_i \cdot X_{bar_i} \qquad L_3 = 3.4417$$

$$\frac{1}{2} \cdot 32.0833333 - \frac{1}{3} \cdot 40.2333333 - \frac{1}{3} \cdot 43.9166667 - \frac{1}{3} \cdot 41.1 + \frac{1}{2} \cdot 58.3 = 3.4417$$

combination four:

$$c4 := \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \qquad L4 := \sum_{i=1}^{k} c4_i \cdot Xbar_i \qquad L4 = -5.4833 \qquad Xbar = \begin{pmatrix} 32.0833 \\ 40.2333 \\ 43.9167 \\ 41.1 \\ 58.3 \end{pmatrix}$$

$$\frac{1}{2} \cdot 32.0833333 - \frac{1}{2} \cdot 40.2333333 - \frac{1}{2} \cdot 43.9166667 + \frac{1}{2} \cdot 41.1 + 0 \cdot 58.3 = -5.4833$$

With Further Conditions as Linear Contrasts:

combination one:

$$\sum c1 = 0$$

combination two:

$$\sum c2 = 0$$

combination three: **< coefficients of the Linear combination must add to zero**

$$\sum c3 = 0$$

combination four:

$$\sum c4 = 0$$

t-Test for Linear Contrasts H_0: L = 0 versus H_1: $L \neq 0$:

Model:

$X_{i,j} = \mu + \alpha_i + \varepsilon_{i,j}$ **< where:**

- **μ is the grand mean of all objects.**
- **α_i is the treatment effect for each class i.**
- **$\varepsilon_{i,j}$ is the error term specific to each object i,j**

$L = c_1 Xbar_1 + c_2 Xbar_2 + c_3 Xbar_3 + ... + c_k Xbar_k$ **< definition of Linear Contrast**

^ where c_i are coefficients of *each* linear contrast

Restrictions:

$\sum_i n_i \cdot \alpha_i := 0$ $\quad$ $\sum_i \alpha_i := 0$ **or** $\alpha_k := 0$ **< restriction for ANOVA α_i's**

$\sum_{i=0}^{k-1} c_i := 0$ **< restriction for the linear contrast**

Assumptions:

ε_{ij} are a random sample ~ $N(0,\sigma^2)$

Hypotheses:

H_0: L = 0 **< Means of Linear Contrast is zero**

H_1: $L \neq 0$ **< Two sided test**

Standard Error of the Linear Contrasts:

$i := 1..k$

combination one:

$$SEL_1 := \sqrt{MS_W \cdot \sum_i \frac{\left(c1_i\right)^2}{n_i}} \qquad SEL_1 = 1.4791$$

combination two:

$$SEL_2 := \sqrt{MS_W \cdot \sum_i \frac{\left(c2_i\right)^2}{n_i}} \qquad SEL_2 = 1.4791$$

combination three:

$$SEL_3 := \sqrt{MS_W \cdot \sum_i \frac{\left(c3_i\right)^2}{n_i}} \qquad SEL_3 = 1.1694$$

combination four:

$$SEL_4 := \sqrt{MS_W \cdot \sum_i \frac{\left(c4_i\right)^2}{n_i}} \qquad SEL_4 = 1.281$$

Test Statistic:

$$L = \begin{pmatrix} -16.55 \\ -9.6667 \\ 3.4417 \\ -5.4833 \end{pmatrix} \qquad SEL = \begin{pmatrix} 1.4791 \\ 1.4791 \\ 1.1694 \\ 1.281 \end{pmatrix}$$

combination one:

$$t_1 := \frac{L_1}{SEL_1} \qquad t_1 = -11.189$$

combination two:

$$t_2 := \frac{L_2}{SEL_2} \qquad t_2 = -6.5354$$

combination three:

$$t_3 := \frac{L_3}{SEL_3} \qquad t_3 = 2.9432$$

combination four:

$$t_4 := \frac{L_4}{SEL_4} \qquad t_4 = -4.2806$$

$$t = \begin{pmatrix} -11.189 \\ -6.5354 \\ 2.9432 \\ -4.2806 \end{pmatrix}$$

Sampling Distribution of the test Statistic t:

If Assumptions hold and H_0 is true then $t \sim t_{(N-k)}$ **where: k = number of classes**
N = total number of observations

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set** **<Note: degrees of freedom = (N-k)**

$C_1 := qt\left(\frac{\alpha}{2}, N-k\right)$ $C_1 = -2.0595$ $C_2 := qt\left(1-\frac{\alpha}{2}, N-k\right)$ $C_2 = 2.0595$

$C := |C_1|$ $C = 2.0595$

Decision Rule:

IF |t| > C, THEN REJECT H_0 OTHERWISE ACCEPT H_0

Probability Values:

all combinations in order from top to bottom:

$$t = \begin{pmatrix} -11.189 \\ -6.5354 \\ 2.9432 \\ -4.2806 \end{pmatrix}$$

$P := 2 \cdot (1 - pt(t, N-k))$ $P = \begin{pmatrix} 2 \\ 1.999999 \\ 0.006918 \\ 1.99976 \end{pmatrix}$ **< if t > 0**

$P := 2 \cdot pt(t, N-k)$ $P = \begin{pmatrix} 3.1702 \times 10^{-11} \\ 7.598 \times 10^{-7} \\ 1.9931 \\ 0.0002 \end{pmatrix}$ **< if $t \le 0$** **< Two sided case**

t-Test Confidence Intervals:

all combinations in order from top to bottom:

$CI := (L - C \cdot SEL \quad L + C \cdot SEL)$ $CI = \left[\begin{pmatrix} -19.5963 \\ -12.713 \\ 1.0333 \\ -8.1215 \end{pmatrix} \begin{pmatrix} -13.5037 \\ -6.6203 \\ 5.85 \\ -2.8451 \end{pmatrix}\right]$ $L = \begin{pmatrix} -16.55 \\ -9.6667 \\ 3.4417 \\ -5.4833 \end{pmatrix}$

Prototype in R:

```
#ONE-WAY ANOVA CONTRASTS

ZAR=read.table("c:/DATA/Biostatistics/ZarEX11.1R.txt")
ZAR
attach(ZAR)
Y=conc
X=factor(code)
options(digits=12)
```

```
#DESCRIPTIVE STATISTICS:
N=length(Y)
N
k=5
#MEANS:
Xbar=tapply(Y,ZAR[,-1],mean)
Xbar
#NUMBERS:
tapply(Y,ZAR[-1],length)
#STANDARD DEVIATIONS:
tapply(Y,ZAR[-1],sd)

#GENERATING ANOVA TABLE:
FM=lm(Y~X)
anova(FM)
```

```
> Xbar
           1            2            3            4            5
32.0833333333 40.2333333333 43.9166666667 41.1000000000 58.3000000000

>code
           1            2            3            4            5
3.20525610001 2.53034911952 3.14478404134 3.66606055596 3.03644529014
```

```
anova(FM)
Analysis of Variance Table

Response: Y
          Df       Sum Sq      Mean Sq  F value     Pr(>F)
X          4 2191.7286667  547.9321667 55.65475 4.3617e-12 ***
Residuals 25  246.1300000    9.8452000
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '
1
```

```
#MAKING CONTRASTS IN R
require(gmodels) #MUST LOAD {gmodels} PACKAGE FROM CRAN
#CONTRAST FOR ZAR Example 11.5
C1=c(0,1/3,1/3,1/3,-1)
fit.contrast(FM,X,C1,conf.int=0.95)
C2=c(1,-1/3,-1/3,-1/3,0)
fit.contrast(FM,X,C2,conf.int=0.95)
C3=c(1/2,-1/3,-1/3,-1/3,1/2)
fit.contrast(FM,X,C3,conf.int=0.95)
C4=c(1/2,-1/2,-1/2,1/2,0)
fit.contrast(FM,X,C4,conf.int=0.95)
```

```
> fit.contrast(FM,X,C4,conf.int=0.95)
                                  Estimate    Std. Error         t value
X c=( 0.5 -0.5 -0.5 0.5 0 ) -5.48333333333 1.28096317928 -4.28063305959
                                     Pr(>|t|)        lower CI        upper CI
X c=( 0.5 -0.5 -0.5 0.5 0 ) 0.000240337514091 -8.12152638571 -2.84514028095
```

Multiple Comparisons Procedures (MCP) for linear contrasts:

Working with multiple linear contrasts makes one subject to the same concerns about planned (*a priori*) versus unplanned (*post hoc*) multiple comparisons as encountered with pairwise comparisons. Thus, it is often standard practice to specify familywise levels of type I error α_{fw} or not - as described in Worksheet 250. Bonferroni (or Holm) estimates are often encountered with α_{fw} determined by the number of planned tests (g). The method of Scheffé provides a means of evaluating any number of linear contrasts *post hoc* since it gives an α_{fw} for all possible linear contrasts. Since the Tukey procedure applies only to pairwise comparisons (i.e., some but not all possible linear contrasts), however, it is not applicable in this more general context.

Bonferroni MCP:

Methodology of the test is the same as above. The only thing that changes is adjustment by Bonferroni's factor g in calculating the critical value of the test, probability, and confidence intervals.

Bonferroni MCP Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set** **<Note: degrees of freedom = (N-k)**

$g := 4$ **< Number of planned (a priori) tests must be explicitly set**

$C_1 := qt\left(\frac{\alpha}{2 \cdot g}, N - k\right)$ $C_1 = -2.6916$ $C_2 := qt\left(1 - \frac{\alpha}{2 \cdot g}, N - k\right)$ $C_2 = 2.6916$

$C_B := |C_1|$ $C_B = 2.6916$

Bonferroni Probability Values:

all combinations in order from top to bottom:

$$t = \begin{pmatrix} -11.189 \\ -6.5354 \\ 2.9432 \\ -4.2806 \end{pmatrix}$$

$P := 2 \cdot g \cdot (1 - pt(t, N - k))$ $P = \begin{pmatrix} 8 \\ 7.999997 \\ 0.027671 \\ 7.999039 \end{pmatrix}$ **< if t > 0**

$P := 2 \cdot g \cdot pt(t, N - k)$ $P = \begin{pmatrix} 1.2681 \times 10^{-10} \\ 3.0392 \times 10^{-6} \\ 7.9723 \\ 0.001 \end{pmatrix}$ **< if** $t \le 0$ **< Two sided case**

Bonferroni Confidence Intervals:

all combinations in order from top to bottom:

$CI := \left(L - C_B \cdot SEL \quad L + C_B \cdot SEL\right)$ $CI = \left[\begin{pmatrix} -20.5312 \\ -13.6478 \\ 0.2943 \\ -8.9311 \end{pmatrix} \begin{pmatrix} -12.5688 \\ -5.6855 \\ 6.5891 \\ -2.0355 \end{pmatrix}\right]$ $L = \begin{pmatrix} -16.55 \\ -9.6667 \\ 3.4417 \\ -5.4833 \end{pmatrix}$

Scheffé Multiple Comparisons Procedure:

This procedure is designed to provide a simultaneous probability of α_{fw} for all possible linear contrasts within the ANOVA data structure. Since all possible Linear Constrasts in a dataset involves an infinite set of possible comparisons including pairwise comparisons, the Tukey procedure will typically give smaller Confidence Intervals for all pairwise comparisons, and the Bonferroni procedure will give smaller Confidence Intervals, for a specific limited set of linear contrasts. Thus the Scheffé is a conservative approach that allows "data snooping" and is often preferred for these reasons - if the data will permit it. Often the data does not. In using this test, many researchers relax the criterion of "acceptable" *familywise* Type I error α_{fw} a little (α_{fw} = 0.1 is often considered acceptable for multiple comparisons when α = 0.05 is considered acceptable for single comparisons).

Methodology:

Scheffé intervals are calculated by constructing an unbiased point estimate of the mean of a Linear Combination of interest L, standard deviation of the linear contrast s_L, and Critical Values calculated from the F distribution. If the ANOVA F-Test for H_0: All $\alpha_i = 0$ rejects H_0 then the Scheffé Procedure is guaranteed to find at least one contrast such that H_0: $L_i = 0$ is also rejected.

Scheffé Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$S := \sqrt{(N-1)\cdot qF(1-\alpha, k-1, N-k)}$ **< where k is the number of Populations in the ANOVA data structure, and N is the total number of observations Σn_i.**

$S = 8.9444$

Scheffé Probability Values:

all combinations in order from top to bottom:

$$t = \begin{pmatrix} -11.189 \\ -6.5354 \\ 2.9432 \\ -4.2806 \end{pmatrix}$$

$$P := 1 - pF\left[\frac{(t)^2}{k-1}, k-1, N-k\right] \qquad P = \begin{pmatrix} 2.107317\times 10^{-9} \\ 0.000035 \\ 0.102302 \\ 0.00651 \end{pmatrix}$$

Scheffé Confidence Interval for Multiple Comparisons:

all combinations in order from top to bottom:

$$CI := (L - S\cdot SEL \quad L + S\cdot SEL) \qquad CI = \left[\begin{pmatrix} -29.7799 \\ -22.8966 \\ -7.0175 \\ -16.9408 \end{pmatrix} \begin{pmatrix} -3.3201 \\ 3.5633 \\ 13.9009 \\ 5.9741 \end{pmatrix}\right] \qquad L = \begin{pmatrix} -16.55 \\ -9.6667 \\ 3.4417 \\ -5.4833 \end{pmatrix}$$

Scheffé Multiple Pairwise Comparison (MCP) of Means:

The Scheffé approach to Linear Combinations can be applied to pairwise comparisons of means. In this case a linear contrast consists of coeffients 1 and -1 for each pair of means compared with all other coefficients set to zero. The section below adds Scheffé pairwise calculations for the same data, and in the same format, as other MCP tests presented in Lecture Worksheet 250.

$k := 3$ **< number of classes**

$N := 18$ **< total number of cases**

Zar Example 10.3

$$X_{bar} := \begin{pmatrix} 26.9833333333 \\ 25.6666666667 \\ 29.55 \end{pmatrix} \quad n := \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix} \quad s := \begin{pmatrix} 0.679460570355 \\ 1.13078144072 \\ 1.26767503722 \end{pmatrix}$$

^ descriptive statistics derived from R

```
Analysis of Variance Table

Response: Y
          Df      Sum Sq     Mean Sq  F value     Pr(>F)
X          2 46.80333333 23.40166667 20.97341 4.5151e-05 ***
Residuals 15 16.73666667  1.11577778
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

One-Way ANOVA Table:

Values taken from R's ANOVA table:

Source:	**SS**	**df**		**MS**	
Between	$SS_B := 46.80333333$	$df_B := k - 1$	$df_B = 2$	$MS_B := \frac{SS_B}{df_B}$	$MS_B = 23.4017$
Within	$SS_W := 16.73666667$	$df_W := N - k$	$df_W = 15$	$MS_W := \frac{SS_W}{df_W}$	$MS_W = 1.1158$
TOTAL	$SS_T := SS_B + SS_W$	$SS_T = 63.54$			

Linear Contrasts:

$$c_1 := \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \qquad L_1 := \sum_{i=1}^{k} c_{1_i} \cdot X_{bar_i} \qquad L_1 = 1.3167$$

< Comparing sample 1 with 2

$\sum c_1 = 0$ **< coefficients must sum to zero**

$$c_2 := \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \qquad L_2 := \sum_{i=1}^{k} c_{2_i} \cdot X_{bar_i} \qquad L_2 = -2.5667$$

< Comparing sample 1 with 3

$\sum c_2 = 0$ **< coefficients must sum to zero**

$$c_3 := \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad L_3 := \sum_{i=1}^{k} c_{3_i} \cdot X_{bar_i} \qquad L_3 = -3.8833$$

< Comparing sample 1 with 3

$\sum c_3 = 0$ **< coefficients must sum to zero**

$$L := \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} \qquad L = \begin{pmatrix} 1.3167 \\ -2.5667 \\ -3.8833 \end{pmatrix}$$

< Linear Contrasts are pairwise differences between means

Standard Error of the Linear Contrasts:

$i := 1..k$ $\qquad MS_W = 1.1158$

$$SEL_1 := \sqrt{MS_W \cdot \sum_i \frac{\left(c_{1_i}\right)^2}{n_i}} \qquad SEL_1 = 0.6099$$

combination one:

$$SEL_2 := \sqrt{MS_W \cdot \sum_i \frac{\left(c_{2_i}\right)^2}{n_i}} \qquad SEL_2 = 0.6099$$

combination two:

$$SEL_3 := \sqrt{MS_W \cdot \sum_i \frac{\left(c_{3_i}\right)^2}{n_i}} \qquad SEL_3 = 0.6099$$

combination three:

$$SEL := \begin{pmatrix} SEL_1 \\ SEL_2 \\ SEL_3 \end{pmatrix} \qquad SEL = \begin{pmatrix} 0.6099 \\ 0.6099 \\ 0.6099 \end{pmatrix}$$

Scheffé-Test for Linear Contrasts H_0: L = 0 versus H_1: $L \neq 0$:

Model:

$X_{i,j} = \mu + \alpha_i + \varepsilon_{i,j}$ < where:
- μ is the grand mean of all objects.
- α_i is the mean of $i = \mu + \alpha_i$ for each class i.
- $\varepsilon_{i,j}$ is the error term specific to each object i,j

$L = c_1 Xbar_1 + c_2 Xbar_2 + c_3 Xbar_3 + \ldots + c_k Xbar_k$ < definition of Linear Contrast

^ where c_i are coefficients of *each* linear contrast

Restrictions:

$\sum_i n_i \cdot \alpha_i := 0$ $\sum_i \alpha_i := 0$ or $\alpha_k := 0$ < restriction for ANOVA α_i's

$\sum_{i=0}^{k-1} c_i := 0$ < restriction for the linear contrast

Assumptions:

ε_{ij} are a random sample ~ $N(0,\sigma^2)$

Hypotheses:

H_0: L = 0 < Means of Linear Contrast is zero

H_1: $L \neq 0$ < Two sided test

Test Statistic:

$S_1 := \frac{L_1}{SEL_1}$ $S_1 = 2.159$ < combination one:

$S_2 := \frac{L_2}{SEL_2}$ $S_2 = -4.2086$ < combination two:

$S_3 := \frac{L_3}{SEL_3}$ $S_3 = -6.3676$ < combination three:

$L = \begin{pmatrix} 1.3167 \\ -2.5667 \\ -3.8833 \end{pmatrix}$ $SEL = \begin{pmatrix} 0.6099 \\ 0.6099 \\ 0.6099 \end{pmatrix}$

$S := \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix}$ $S = \begin{pmatrix} 2.159 \\ -4.2086 \\ -6.3676 \end{pmatrix}$

Sampling Distribution of the test Statistic S:

If Assumptions hold and H_0 is true then S ~$F_{(k-1,N-k)}$ where: k = number of classes

N = total number of observations

$k = 3$

$N = 18$

Scheffé Critical Value of the Test:

$\alpha := 0.05$ < Probability of Type I error must be explicitly set

$C_S := \sqrt{(k-1) \cdot qF(1-\alpha, k-1, N-k)}$ < where k is the number of Populations in the ANOVA data structure, and N is the total number of observations Σn_i.

$C_S = 2.7138$

Scheffé Probability Values:

all combinations in order from top to bottom:

$$S = \begin{pmatrix} 2.159 \\ -4.2086 \\ -6.3676 \end{pmatrix}$$

$$P := 1 - pF\left[\frac{(S)^2}{k-1}, k-1, N-k\right] \qquad P = \begin{pmatrix} 0.13140556 \\ 0.00288616 \\ 0.00005442 \end{pmatrix}$$

Scheffé Confidence Interval for Multiple Comparisons:

$$SEL = \begin{pmatrix} 0.6099 \\ 0.6099 \\ 0.6099 \end{pmatrix}$$

all combinations in order from top to bottom:

$$CI := \left(L - C_S \cdot SEL \quad L + C_S \cdot SEL\right) \qquad CI = \left[\begin{pmatrix} -0.3384 \\ -4.2217 \\ -5.5384 \end{pmatrix} \begin{pmatrix} 2.9717 \\ -0.9116 \\ -2.2283 \end{pmatrix}\right] \qquad L = \begin{pmatrix} 1.3167 \\ -2.5667 \\ -3.8833 \end{pmatrix}$$

Prototype in Systat:

```
Using model MSE of 1.116 with 15 df.
Matrix of pairwise mean differences:

                      1           2           3
           1     0.00bn000000
           2    -1.31666667  0.00000000
           3     2.56666667  3.88333333  0.00000000

Scheffe Test.
Matrix of pairwise comparison probabilities:

                      1           2           3
           1     1.00000000
           2     0.13140556  1.00000000
           3     0.00288616  0.00005442  1.00000000
```

Note: Prototype run from this data in SYSTAT. R lacks a convenient way to construct *pairwise* Scheffé tests. Because Scheffé *post-hoc* testing is very conservative, other pairwise methods are usually preferable.

1