ORIGIN ≡ 1

Bartlett's Test for Homogeneity of Variance

Bartlett's Test is designed to test for *homoscedasticity* - homogeneity of variance. Homoscedascity is an underlying assumption of ANOVA.

Model:

$X_{i,j} = \mu + \alpha_i + \varepsilon_{i,j}$ < where:

μ is the grand mean of all objects.
α_i is the treatment where mean for each class is $\mu + \alpha_i$.
$\varepsilon_{i,j}$ is the error term specific to each object i,j

Restriction:

$\sum_i n_i \cdot \alpha_i := 0$ **or** $\sum_i \alpha_i := 0$ **or** $\alpha_k := 0$ **< allows estimation of k parameters. See Rosner 2006 p. 558**

Assumptions:

ε_{ij} are a random sample ~ $N(0,\sigma^2)$

< Zar's Example 10.13

```
#BARTLETT'S TEST OF VARIANCE HOMOGENEITY
#ZAR EXAMPLE 10.13
ZAR=read.table("c:/DATA/Biostatistics/ZarEX10.13R.txt")
ZAR
attach(ZAR)
options(digits=12)
s1=sqrt(var(weights[feed=="1"]))
s1
s2=sqrt(var(weights[feed=="2"]))
s2
s3=sqrt(var(weights[feed=="3"]))
s3
s4=sqrt(var(weights[feed=="4"]))
s4
anova(lm(weights~factor(feed)))
```

```
> s1
[1] 3.34544466402
> s2
[1] 3.06838719851
> s3
[1] 3.41418609139
> s4
[1] 2.41619535634
```

$k := 4$ $\quad i := 1..4$ $\quad n := \begin{pmatrix} 5 \\ 5 \\ 4 \\ 5 \end{pmatrix}$ $\quad N := \sum n$ $\quad N = 19$ $\quad \nu := n - 1$ $\quad \nu = \begin{pmatrix} 4 \\ 4 \\ 3 \\ 4 \end{pmatrix}$

$s := \begin{pmatrix} 3.34544466402 \\ 3.06838719851 \\ 3.41418609139 \\ 2.41619535634 \end{pmatrix}$ $\quad s^2 = \begin{pmatrix} 11.192 \\ 9.415 \\ 11.6567 \\ 5.838 \end{pmatrix}$ $\quad \sum_i \nu_i \cdot \log\left[\left(s_i\right)^2\right] = 14.3557$ $\quad \sum_i \frac{1}{\nu_i} = 1.0833$ $\quad \sum_i \nu_i = 15$

```
Analysis of Variance Table

Response: weights
             Df      Sum Sq     Mean Sq F value      Pr(>F)
factor(feed)  3 338.9373684 112.9791228 12.0404 0.00028301 ***
Residuals    15 140.7500000   9.3833333
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

< MS_W

pooled variance:

$MS_W = s_p^{\,2}$

$s_p := \sqrt{9.3833333}$

Hypotheses:

H_0: σ_i^2 are the same *for all* i

H_1: *At least one* σ_i^2 not the same as the others < **One sided test**

Test Statistic:

$$B := \ln\left(s_p^2\right) \cdot \left(\sum_i \nu_i\right) - \sum_i \nu_i \cdot \ln\left[\left(s_i\right)^2\right] \qquad B = 0.5288$$

$$B := 2.30259 \cdot \left[\log\left(s_p^2\right) \cdot \left(\sum_i \nu_i\right) - \sum_i \nu_i \cdot \log\left[\left(s_i\right)^2\right]\right] \qquad B = 0.5288$$

Correction factor:

$$C := 1 + \frac{1}{3 \cdot (k-1)} \cdot \left[\sum_i \frac{1}{\nu_i} - \frac{1}{\left(\sum_i \nu_i\right)}\right] \qquad C = 1.113$$

Corrected Test Statistic:

$$B_c := \frac{B}{C} \qquad B_c = 0.4752$$

Sampling Distribution of the test Statistic B_c:

If Assumptions hold and H_0 is true then $B_c \sim \chi^2_{(k-1)}$

where: k = number of classes
N = total number of observations

Critical Value of the Test:

$\alpha := 0.05$ < **Probability of Type I error must be explicitly set**

$$C := \text{qchisq}(1 - \alpha, k - 1) \qquad C = 7.8147$$

Decision Rule:

IF B_c > C, THEN REJECT H_0, OTHERWISE ACCEPT H_0

Probability Value:

$$P := 1 - \text{pchisq}(B_c, k - 1) \qquad P = 0.924316$$

Prototype in R:

```
#BARTLETT'S TEST:
bartlett.test(weights,feed)
```

```
        Bartlett test of homogeneity of variances

data:  weights and feed
Bartlett's K-squared = 0.4752, df = 3, p-value = 0.9243
```

^ B_c