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Multi-Factor ANOVA Models

There are many interesting and powerful variants of ANOVA involving two or more *factors*. In all ANOVA situations, "factor" is the term one applies to general descriptors of groups. Within each factor there generally two or more "levels" - again descriptors of groups that differ in some meaningful way related to a factor. For instance, in setting up groups, "sex" might be a factor with "male" and "female" representing levals within factor sex. It should be noted that all factors and levels are only a means of placing observations within one group or another. Thus, they never have numerical value, although descriptor *names* may be "1", "2", etc., and when using R we must remember to say so by using the function factor(). In the literature, one often encounters the term "factorial ANOVA". Such a term *may* indicate that *multiple* factors are *somehow* involved, as in a two-way or higher experimental design. I generally shy away from using this term however, because all ANOVA's are by this definition "factorial".

As for the *meaning* of factors and levels, it is generally agreed that there two kinds:

- Fixed factor:

Here, a factor has a *limited number* of specific levels, and means of individuals within a level of a factor has meaning and invites comparisons with means from other levels within the same factor. This situation is commonly encountered in *observational data* such as factor "sex" with only two levels "male" and "female". It is also common with *expermimental data* such as testing different and specific concentrations (levels) of an experimental drug (factor).

- Random factor:

Here, although a factor may have a limited number of levels within a particular study, it is judged that the levels chosen are *only examples* of a much larger, perhaps infinite, set of levels that might have been chosen for the study with equal reason. For example, four different sets of 10 mice each might represent levels within some "subject" factor, but these animals only represent random groupings out of a much larger set of mice that might have been chosen for the study. In this case, differences in means between such levels carry little useful information. Random factors are commonly encountered in both observational and experimental data.

As a practical matter, it is often easy to tell the difference between fixed and random factors. In other situations, however, the difference is not so clear. In setting up an experiment or set of observations sometimes one has a choice.

Depending upon whether an ANOVA study contains fixed or random factors or both, Zar 2010 p. 262 presents a standard classification of what's he terms "ANOVA Models" although in the literature terminology varies. As a cautionary note, the word "model" appears in statistical literature in many different and generally unrelated contexts. Here we talk about "ANOVA Type I, II, or III Models". In another context, discussion may center on "ANOVA contrast models", and in Regression one may encounter "Regression Sum of Squares Models I-III" and so forth. There may be a deep underlying mathematical connection between "models" in these different contexts, but that's generally not what authors are trying to convey.

ANOVA Model I:

Also called a "fixed effects" model, all factors are fixed factors. It is therefore generally useful to look at multiple questions concerning levels and interactions using multiple F-tests.

ANOVA Model II:

Also called a "random effects" or "pure random effects" model, all factors are random factors. Use of such tests is somewhat limited, although it is often of interest to ask how much of measured variance can be attributed to one factor versus another.

ANOVA Model III:

Also called a "mixed effects" model or just a "mixed" model, some factors are fixed whereas others are random. Here, it is generally useful to compare means between levels of fixed factors only. Additionally, one may be interested in the amount of variance attributable to one or more random factors either as a matter of direct interest, or often as a means of controlling for variance in an experiment that one is not interested in studying. In the latter context, a random factor may be called a "nuisance" factor. There are several standard and very useful ANOVA experimental designs of this type, including "Randomized Block", "Repeated Measures" and "Latin Squares" ANOVA designs.

Statistics and Decision Rules for ANOVA Type I,II,III Models:

In all three ANOVA models, calculation of the ANOVA Table is the same. In statistical computer programs, F statistics are typically given along with Probability (P) values for Type I ANOVA only. Critical Values (C) are obtained from qF() with numerator and denominator degrees of freedom corresponding to the the *numerator* Mean Squares and *denominator* Mean Squares used to calculate the F ratio. Before using new software, it is wise to verify these calculations.

For Type II and III ANOVA, one will usually need to calculate appropriate F ratios and Critical Values by hand, following the table below (from Zar 2010 Table 12.3 p.262):

Hypothesized Effect	Model I ANOVA factors A & B fixed	Model II ANOVA factors A & B random	Model III ANOVA factor A fixed Factor B random
Factor A	$\frac{MS_A}{MS_E}$	$\frac{MS_A}{MS_{AB}}$	$\frac{MS_A}{MS_{AB}}$
Factor B	$\frac{MS_B}{MS_E}$	$\frac{MS_B}{MS_{AB}}$	$\frac{MS_B}{MS_E}$
A X B Interaction	$\frac{MS_{AB}}{MS_E}$	$\frac{MS_{AB}}{MS_E}$	$\frac{MS_{AB}}{MS_E}$

Note: all of the above apply to ANOVA studies with "balanced" data - equal numbers of observations in each level for all factors. Unbalanced ANOVA designs are more complex and often involve a General Linear models (GLM) approach. Even here, unbalanced data often create complications.