ORIGIN ≡ 1

Nested Two-way ANOVA Model for Balanced Data

Nested ANOVA designs occur when, either by efficiency of design or by necessity, not all combinations of factors are studied. Nested design in a two-way ANOVA (involving two factors) permits analysis of the independent factor (factor A below) in the same way as a standard (crossed) two-way ANOVA. The difference lies in factor B nested within A. Here effects of different cell blocks within single treatment levels of A are assessed much as if in single factor ANOVA.

Data Structure:

Data are structured such that different levels of Nested Factor B occur only within single levels of independent Factor A. Other possible combinations are not studied.

Let index i,j indicate the ith row (treatment classes of Variable A) and jth column (treatment classes of Variable B)

Treatment Classes of Variable A:	Nested Two-Way ANOVA					
	Treatment Classes of Variable B Nested Within A:					
	#1	#2	#3	#4	#5	#6
#1	n	n				
#2			n	n		
#3					n	n

Each cell consists of n replicates with means $Ybar_{ij}$

Also let:
- $Ybar_{i.}$ = mean over all columns for A_i.
- $Ybar_{ij}$ = mean within data blocks.
- $Ybar_{..}$ = overall mean.

Model:

Note: this model has no associated test for interactions because of the nested design.

$$Y_{i,jk} = \mu + \alpha_i + \beta_{j(i)} + \varepsilon_{ijk}$$

where:

- μ **is a constant = grand mean of all objects.**
- α_i **are effect coefficients for classes i in Variable A.**
- β_j **are nested coefficients for classes j of Variable B in A.**
- ε_{ijk} **is the error term specific to each object i,j,k**
  i = 1 to a, j = 1 to b, k = 1 to n, N = abn

Restrictions:

$$\sum_i \alpha_i := 0 \qquad \sum_j \beta_{j\_in\_i} := 0$$

Assumptions:

- ε_{ijk} **are a random sample ~ $N(0,\sigma^2)$**
- **variance is homogeneous across cells**

Variable Assignment:

ZAR := READPRN("c:/DATA/Biostatistics/ZarEX15.1aR.txt")

$X := ZAR^{\langle 2 \rangle}$ **< response (dependent) variable Y**

Zar Example 15.1

	conc	drug	source
1	102	1	A
2	104	1	A
3	103	1	Q
4	104	1	Q
5	108	2	D
6	110	2	D
7	109	2	B
8	108	2	B
9	104	3	L
10	106	3	L
11	105	3	S
12	107	3	S

B_1 B_2

A_1 $\quad Y_{11} := \begin{pmatrix} 102 \\ 104 \end{pmatrix} \quad Y_{12} := \begin{pmatrix} 103 \\ 104 \end{pmatrix}$

B_3 B_4

A_2 $\quad Y_{21} := \begin{pmatrix} 108 \\ 110 \end{pmatrix} \quad Y_{22} := \begin{pmatrix} 109 \\ 108 \end{pmatrix}$ **< response blocks**

B_5 B_6

A_3 $\quad Y_{31} := \begin{pmatrix} 104 \\ 106 \end{pmatrix} \quad Y_{32} := \begin{pmatrix} 105 \\ 107 \end{pmatrix}$

a := 3 b := 2 n := 2 N := 12

Means:

$GM := \text{mean}(X)$ $GM = 105.8333$ **< grand mean**

$$B := \begin{pmatrix} \text{mean}(Y_{11}) & \text{mean}(Y_{12}) \\ \text{mean}(Y_{21}) & \text{mean}(Y_{22}) \\ \text{mean}(Y_{31}) & \text{mean}(Y_{32}) \end{pmatrix} \qquad B = \begin{pmatrix} 103 & 103.5 \\ 109 & 108.5 \\ 105 & 106 \end{pmatrix}$$

< block means

$$A := \begin{pmatrix} \text{mean}(Y_{11}, Y_{12}) \\ \text{mean}(Y_{21}, Y_{22}) \\ \text{mean}(Y_{31}, Y_{32}) \end{pmatrix} \qquad A = \begin{pmatrix} 103.25 \\ 108.75 \\ 105.5 \end{pmatrix}$$

< Independent Factor A means (row means)

Sums of Squares:

$k := 1 .. N$

$$SSTO := \sum_k \left(X_k - GM\right)^2 \qquad SSTO = 71.6667$$

< total SS

$i := 1 .. a$

$$SSA := b \cdot n \cdot \sum_{i=1}^{a} \left(A_i - GM\right)^2 \qquad SSA = 61.1667$$

< among groups SS

$i := 1 .. b$

$$SSB := n \cdot \sum_{i=1}^{a} \sum_{j=1}^{b} \left(B_{i,j} - GM\right)^2 \qquad SSB = 62.6667$$

< among all subgroups SS

$SSBinA := SSB - SSA$ $SSBinA = 1.5$ **< subgroups SS**

$SSE := SSTO - SSB$ $SSE = 9$ **< error SS**

ANOVA Table for Nested design:

Sum of Squares:	Degrees of Freedom:		Mean Squares:	
$SSA = 61.1667$	$df_A := a - 1$	$df_A = 2$	$MSA := \frac{SSA}{df_A}$	$MSA = 30.5833$
$SSBinA = 1.5$	$df_{BinA} := a \cdot (b - 1)$	$df_{BinA} = 3$	$MSBinA := \frac{SSBinA}{df_{BinA}}$	$MSBinA = 0.5$
$SSE = 9$	$df_E := a \cdot b \cdot (n - 1)$	$df_E = 6$	$MSE := \frac{SSE}{df_E}$	$MSE = 1.5$
$SSTO = 71.6667$	$dfT := N - 1$	$dfT = 11$		

F-Test for Independent Factor Effects:

Hypotheses:

H_0: $\alpha_i = 0$ *for all* i

H_1: *At least one* $\alpha_i \neq 0$

Test Statistic:

$F := \frac{MSA}{MSE}$ $F = 20.3889$

Sampling Distribution of the Test Statistic F:

If Assumptions hold and H_0 is true then $F \sim F_{(dfA,dfE)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C := qF(1 - \alpha, df_A, df_E)$ $C = 5.1433$

Decision Rule:

IF F > C, THEN REJECT H_0 OTHERWISE ACCEPT H_0

Probability Value:

$P := 1 - pF(F, df_A, df_E)$ $P = 0.00211$

F-Test for Nested Factor Specific Effects:

Hypotheses:

H_0: $\beta_{j\ in\ i} = 0$ *for all values of j within a specific level i*

H_1: *At least one* $\beta_{j\ in\ i} <> 0$

Test Statistic:

$F := \frac{MSBinA}{MSE}$ $F = 0.3333$

Sampling Distribution of the Test Statistic F:

If Assumptions hold and H_0 is true then $F \sim F_{(dfBinA,dfE)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C := qF(1 - \alpha, df_{BinA}, df_E)$ $C = 4.7571$

Decision Rule:

IF F > C, THEN REJECT H_0 OTHERWISE ACCEPT H_0

Probability Value:

$P := 1 - pF(F, df_{BinA}, df_E)$ $P = 0.802202$

Prototype in R:

```
#BALANCED NESTED ANOVA
#FIXED EFFECTS (ANOVA MODEL I)
#ZAR EXAMPLE 15.1a
ZAR=read.table("c:/DATA/Biostatistics/ZarEX15.1aR.txt")
ZAR
attach(ZAR)
Y=conc
A=factor(drug)
B=factor(source)

anova(lm(Y~A+B%in%A))
```

```
Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value   Pr(>F)
A          2 61.167  30.583 20.3889 0.002110 **
A:B        3  1.500   0.500  0.3333 0.802202
Residuals  6  9.000   1.500
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Confidence Intervals for Fixed Factor Means:

From calculations above:

$$A = \begin{pmatrix} 103.25 \\ 108.75 \\ 105.5 \end{pmatrix}$$ **< factor A means**

$a = 3$

$b = 2$

Critical Value:

$$C := \left| qt\left(\frac{\alpha}{2}, df_E\right) \right| \qquad C = 2.4469$$

For Factor A:

$$CI_A := \left(A - C \cdot \sqrt{\frac{MSE}{b \cdot n}} \quad A + C \cdot \sqrt{\frac{MSE}{b \cdot n}} \right) \qquad CI_A = \left[\begin{pmatrix} 101.7516 \\ 107.2516 \\ 104.0016 \end{pmatrix} \begin{pmatrix} 104.7484 \\ 110.2484 \\ 106.9984 \end{pmatrix} \right]$$