ORIGIN ≡ 1

Cross-Nested Two-way ANOVA Model for Balanced Data

This example is an extension of Crossed and Nested Design in which there are two Cross factors A & B, plus nested factor C within A. This example from p. 1149-1154 in Neter et al. 1990 *Applied Linear Statistical Models 4th Edition* is designed to show one of many sophisticated ANOVA models that can be constructed. Once designed, each term of the model can be tested for significance (i.e., not being zero) by a corresponding F-test.

Data Structure:

NKNW Table 28.12

	Factor A			
	A1		A2	
	Factor C in A			
Factor B	C1	C2	C3	C4
B1	16	14	7	4
	20	19	5	9
B2	21	28	11	12
	25	19	17	15

Model:

$$Y_{i,j,k,m} = \mu + \alpha_i + \beta_j + \gamma_{k(i)} + \alpha\beta_{ij} + \beta\gamma_{jk(i)} + \varepsilon_{ijkm}$$

Restrictions:

$\sum_i \alpha_i := 0$ $\sum_i \beta_j := 0$

$\sum_i \alpha\beta_{ij} := 0$ $\sum_i \beta\gamma_{jk(i)} := 0$

where:

μ is a constant = grand mean of all objects.
α_i are effect coefficients for classes i in Variable A.
β_j are effect coefficients for classes j in Variable B.
$\gamma_{k(i)}$ are nested effects of Variable C in A.
$\alpha\beta_{ij}$ are interaction effects of fixed Variables A & B.
$\beta_{\gamma jk(i)}$ are interaction effects of fixed Variable B with C inside A
ε_{ijkm} is the error term specific to each object i,j,k,m
i = 1 to a, j = 1 to b, k = 1 to c, m=1 to N = total.

Assumptions:

– **$\gamma_{k(i)}$ are a random sample ~ $N(0,\sigma_\gamma^2)$**
– **$\beta\gamma_{k(i)}$ are a random sample ~ $N(0,\sigma_{\beta\gamma}^2)$**
– **ε_{ijkm} are a random sample ~ $N(0,\sigma^2)$**
– **variables pairwise independent**

ANOVA Table:

Calculations in R (Note formula used here):

```
#THREE FACTOR DESIGN WITH
#ONE FACTOR NESTED INSIDE ANOTHER
#FACTORS A & B ARE CROSSED FACTORS
#FACTOR B IS NESTED WITHIN A
#EXAMPLE FROM NETER ET AL 1990 P. 1152
NET=read.table("c:/DATA/Biostatistics/NKNWTA28.12.txt")
NET
attach(NET)
A=factor(A)
B=factor(B)
C=factor(CinA)
#ANOVA TABLE:
anova(lm(Y~A*B+B:C%in%A+C%in%A))
```

```
Analysis of Variance Table

Response: Y
          Df Sum Sq Mean Sq F value    Pr(>F)
A          1 420.25  420.25 31.7170 0.0004917 ***
B          1 182.25  182.25 13.7547 0.0059672 **
A:B        1   2.25    2.25  0.1698 0.6911001
A:C        2   0.50    0.25  0.0189 0.9813525
A:B:C      2   2.50    1.25  0.0943 0.9109709
Residuals  8 106.00   13.25
```

ANOVA Table:

Values derived from R's ANOVA table

SS	df	MS	
$SSA := 420.25$	$df_A := 1$	$MSA := \frac{SSA}{df_A}$	$MSA = 420.25$
$SSB := 182.25$	$df_B := 1$	$MSB := \frac{SSB}{df_B}$	$MSB = 182.25$
$SSAB := 2.25$	$df_{AB} := 1$	$MSAB := \frac{SSAB}{df_{AB}}$	$MSAB = 2.25$
$SSCinA := 0.50$	$df_{CinA} := 2$	$MSCinA := \frac{SSCinA}{df_{CinA}}$	$MSCinA = 0.25$
$SSBCinA := 2.50$	$df_{BCinA} := 2$	$MSBCinA := \frac{SSBCinA}{df_{BCinA}}$	$MSBCinA = 1.25$
$SSE := 106.00$	$df_E := 8$	$MSE := \frac{SSE}{df_E}$	$MSE = 13.25$

Note: using the ANOVA table for the following tests involves creating proper F-statistic ratios of Mean Squares. This is done by consulting a chart as in NKNW Table 28.11 and looking for algebraic expectations of *Expected Mean Squares*. The ratio is then constructed by using the appropriate Expected Mean Squares as denominator that contains all but the most specific term of the numerator Expected Mean Squares. NKNW Appendix contains general rules for doing this with balanced data, and constructing other kinds of ANOVA models.

F-Test for Independent Factor A Effects:

Hypotheses:

H_0: $\alpha_i = 0$ ***for all*** **i**

H_1: ***At least one*** $\alpha_i \neq 0$

Test Statistic:

$F_A := \frac{MSA}{MSCinA}$ $F_A = 1681$

Sampling Distribution of the Test Statistic F_A:

If Assumptions hold and H_0 is true then $F_A \sim F_{(dfA,DfCinA)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C_A := qF(1-\alpha, df_A, df_{CinA})$ $C_A = 18.5128$

Decision Rule:

IF $F_A > C_A$, THEN REJECT H_0 OTHERWISE ACCEPT H_0

Probability Value:

$P_A := 1 - pF(F_A, df_A, df_{CinA})$ $P_A = 0.000594$

F-Test for Independent Factor B Effects:

Hypotheses:

H_0: $\beta_j = 0$ *for all* j

H_1: *At least one* $\beta_j \neq 0$

Test Statistic:

$F_B := \frac{MSB}{MSBCinA}$ $F_B = 145.8$

Sampling Distribution of the Test Statistic F_B:

If Assumptions hold and H_0 is true then $F_B \sim F_{(dfB,DfBCinA)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C_B := qF(1 - \alpha, df_A, df_{BCinA})$ $C_B = 18.5128$

Decision Rule:

IF $F_B > C_B$, THEN REJECT H_0 OTHERWISE ACCEPT H_0

Probability Value:

$P_B := 1 - pF(F_B, df_B, df_{BCinA})$ $P_B = 0.0067889$

F-Test for Interaction of Independent Factors A & B:

Hypotheses:

H_0: $\alpha\beta_{ij} = 0$ *for all* i

H_1: *At least one* $\alpha\beta_{ij} \neq 0$

Test Statistic:

$F_{AB} := \frac{MSAB}{MSBCinA}$ $F_{AB} = 1.8$

Sampling Distribution of the Test Statistic F_A:

If Assumptions hold and H_0 is true then $F_{AB} \sim F_{(dfAB,DfBCinA)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C_{AB} := qF(1 - \alpha, df_{AB}, df_{BCinA})$ $C_{AB} = 18.5128$

Decision Rule:

IF $F_{AB} > C_{AB}$, THEN REJECT H_0 OTHERWISE ACCEPT H_0

Probability Value:

$P_{AB} := 1 - pF(F_{AB}, df_{AB}, df_{BCinA})$ $P_{AB} = 0.3117528$

F-Test for Nested Factor C Specific Effects:

Hypotheses:

H_0: $\gamma_{k(i)} = 0$ ***for all values of j within specific level k in i***

H_1: ***At least one*** $\gamma_{k(i)} <> 0$

Test Statistic:

$F_C := \frac{MSCinA}{MSE}$ $F_C = 0.0188679$

Sampling Distribution of the Test Statistic F_C:

If Assumptions hold and H_0 is true then $F_C \sim F_{(dfCinA,DfE)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C_C := qF(1 - \alpha, df_{CinA}, df_E)$ $C_C = 4.45897$

Decision Rule:

IF $F_C > C_C$, THEN REJECT H_0 OTHERWISE ACCEPT H_0

Probability Value:

$P_C := 1 - pF(F_C, df_{CinA}, df_E)$ $P_C = 0.9813525$

F-Test for Interaction of Fixed Factor B with Nested Factor C Specific Effects:

Hypotheses:

H_0: $\beta\gamma_{jk(i)} = 0$ ***for all values of j within specific levels j & k in i***

H_1: ***At least one*** $\beta\gamma_{jk(i)} <> 0$

Test Statistic:

$F_{BC} := \frac{MSBCinA}{MSE}$ $F_{BC} = 0.09433962$

Sampling Distribution of the Test Statistic F_{BC}:

If Assumptions hold and H_0 is true then $F_{BC} \sim F_{(dfBCinA,DfE)}$

Critical Value of the Test:

$\alpha := 0.05$ **< Probability of Type I error must be explicitly set**

$C_{BC} := qF(1 - \alpha, df_{BCinA}, df_E)$ $C_{BC} = 4.45897$

Decision Rule:

IF $F_{BC} > C_{BC}$, THEN REJECT H_0 OTHERWISE ACCEPT H_0

Probability Value:

$P_{BC} := 1 - pF(F_{BC}, df_{BCinA}, df_E)$ $P_{BC} = 0.9109709$

Prototype in R:

```
#FROM ANOVA TABLE:
MSA=420.25
MSB=182.25
MSAB=2.25
MSCinA=0.25
MSBCinA=1.25
MSE=13.25
dfA=1
dfB=1
dfAB=1
dfCinA=2
dfBCinA=2
dfE=8

alpha=0.05

#TEST FOR INDEPENDENT FACTOR A:
FA=MSA/MSCinA
FA
CA=qf(1-alpha,dfA,dfCinA)
CA
PA=1-pf(FA,dfA,dfCinA)
PA
#TEST FOR INDEPENDENT FACTOR B:
FB=MSB/MSBCinA
FB
CB=qf(1-alpha,dfB,dfBCinA)
CA
PB=1-pf(FB,dfB,dfBCinA)
PB
#TEST FOR INTERACTION OF FACTORS A & B:
FAB=MSAB/MSBCinA
FAB
CAB=qf(1-alpha,dfAB,dfBCinA)
CAB
PAB=1-pf(FAB,dfAB,dfBCinA)
PAB
#TEST FOR NESTED FACTOR C SPECIFIC EFFECTS:
FC=MSCinA/MSE
FC
CC=qf(1-alpha,dfCinA,dfE)
CC
PC=1-pf(FC,dfCinA,dfE)
PC
#TEST FOR INTERACTION OF INDEPENDENT FACTOR B WITH
#NESTED FACTOR C IN A:
FBC=MSBCinA/MSE
FBC
CBC=qf(1-alpha,dfBCinA,dfE)
CBC
PBC=1-pf(FBC,dfBCinA,dfE)
PBC
```